OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4751/01**

MATHEMATICS (MEI)

Introduction to Advanced Mathematics (C1)

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

Section A (36 marks)

1 Make v the subject of the formula $E = \frac{1}{2}mv^2$. [3]

2 Factorise and hence simplify $\dfrac{3x^2 - 7x + 4}{x^2 - 1}$. [3]

3 **(i)** Write down the value of $\left(\frac{1}{4}\right)^0$. [1]

(ii) Find the value of $16^{-\frac{3}{2}}$. [3]

4 Find, algebraically, the coordinates of the point of intersection of the lines $y = 2x - 5$ and $6x + 2y = 7$. [4]

5 **(i)** Find the gradient of the line $4x + 5y = 24$. [2]

(ii) A line parallel to $4x + 5y = 24$ passes through the point $(0, 12)$. Find the coordinates of its point of intersection with the x-axis. [3]

6 When $x^3 + kx + 7$ is divided by $(x - 2)$, the remainder is 3. Find the value of k. [3]

7 **(i)** Find the value of ${}^8C_3$. [2]

(ii) Find the coefficient of x^3 in the binomial expansion of $\left(1 - \frac{1}{2}x\right)^8$. [2]

8 **(i)** Write $\sqrt{48} + \sqrt{3}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. [2]

(ii) Simplify $\dfrac{1}{5 + \sqrt{2}} + \dfrac{1}{5 - \sqrt{2}}$. [3]

9 **(i)** Prove that 12 is a factor of $3n^2 + 6n$ for all even positive integers n. [3]

(ii) Determine whether 12 is a factor of $3n^2 + 6n$ for all positive integers n. [2]

Section B (36 marks)

10 **(i)**

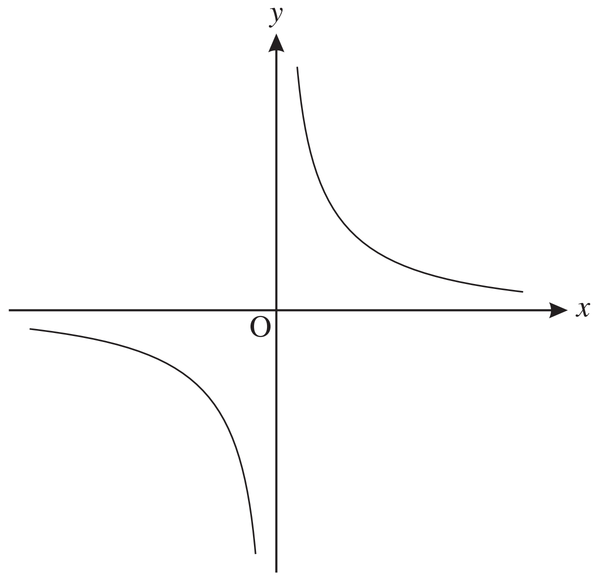

Fig. 10

Fig. 10 shows a sketch of the graph of $y = \frac{1}{x}$.

Sketch the graph of $y = \frac{1}{x-2}$, showing clearly the coordinates of any points where it crosses the axes. [3]

(ii) Find the value of x for which $\frac{1}{x-2} = 5$. [2]

(iii) Find the x-coordinates of the points of intersection of the graphs of $y = x$ and $y = \frac{1}{x-2}$. Give your answers in the form $a \pm \sqrt{b}$.

Show the position of these points on your graph in part **(i)**. [6]

11 **(i)** Write $x^2 - 5x + 8$ in the form $(x-a)^2 + b$ and hence show that $x^2 - 5x + 8 > 0$ for all values of x. [4]

(ii) Sketch the graph of $y = x^2 - 5x + 8$, showing the coordinates of the turning point. [3]

(iii) Find the set of values of x for which $x^2 - 5x + 8 > 14$. [3]

(iv) If $\mathrm{f}(x) = x^2 - 5x + 8$, does the graph of $y = \mathrm{f}(x) - 10$ cross the x-axis? Show how you decide. [2]

[Question 12 is printed overleaf.]

12 A circle has equation $x^2 + y^2 - 8x - 4y = 9$.

(i) Show that the centre of this circle is C (4, 2) and find the radius of the circle. [3]

(ii) Show that the origin lies inside the circle. [2]

(iii) Show that AB is a diameter of the circle, where A has coordinates (2, 7) and B has coordinates (6, −3). [4]

(iv) Find the equation of the tangent to the circle at A. Give your answer in the form $y = mx + c$. [4]

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Section A

1	$[v =][\pm]\sqrt{\frac{2E}{m}}$ www	3	M2 for $v^2 = \frac{2E}{m}$ or for $[v =][\pm]\sqrt{\frac{E}{\frac{1}{2}m}}$ or M1 for a correct constructive first step and M1 for $v = [\pm]\sqrt{k}$ ft their $v^2 = k$; if M0 then SC1 for √*E*/ ½ *m* or √2*E*/*m* etc	3
2	$\frac{3x-4}{x+1}$ or $3 - \frac{7}{x+1}$ www as final answer	3	M1 for $(3x - 4)(x - 1)$ and M1 for $(x + 1)(x - 1)$	3
3	(i) 1 (ii) 1/64 www	1 3	 M1 for dealing correctly with each of reciprocal, square root and cubing (allow 3 only for 1/64) eg M2 for 64 or −64 or 1/√4096 or $\frac{1}{4}^3$ or M1 for $1/16^{3/2}$ or 4^3 or -4^3 or 4^{-3} etc	4
4	$6x + 2(2x - 5) = 7$ $10x = 17$ $x = 1.7$ o.e. isw $y = -1.6$ o.e .isw	M1 M1 A1 A1	for subst or multn of eqns so one pair of coeffts equal (condone one error) simplification (condone one error) or appropriate addn/subtn to eliminate variable allow as separate or coordinates as requested graphical soln: M0	4
5	(i) −4/5 or −0.8 o.e. (ii) (15, 0) or 15 found www	2 3	M1 for 4/5 or 4/−5 or 0.8 or −4.8/6 or correct method using two points on the line (at least one correct) (may be graphical) or for $-0.8x$ o.e. M1 for y = their (i) $x + 12$ o.e. or $4x + 5y = k$ and (0, 12) subst and M1 for using $y = 0$ eg $-12 = -0.8x$ or ft their eqn or M1 for given line goes through (0, 4.8) and (6, 0) and M1 for 6 × 12/4.8 graphical soln: allow M1 for correct required line drawn and M1 for answer within 2mm of (15, 0)	5

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6	f(2) used $2^3 + 2k + 7 = 3$ $k = -6$	M1 M1 A1	or division by $x - 2$ as far as $x^2 + 2x$ obtained correctly or remainder $3 = 2(4 + k) + 7$ o.e. 2nd M1 dep on first	3
7	(i) 56 (ii) −7 or ft from −their (i)/8	2 2	M1 for $\frac{8\times7\times6}{3\times2\times1}$ or more simplified M1 for 7 or ft their (i)/8 or for $56 \times (-1/2)^3$ o.e. or ft; condone x^3 in answer or in M1 expression; 0 in qn for just Pascal's triangle seen	4
8	(i) $5\sqrt{3}$ (ii) common denominator = $(5 - \sqrt{2})(5 + \sqrt{2})$ =23 numerator = 10	2 M1 A1 B1	M1 for $\sqrt{48} = 4\sqrt{3}$ allow M1A1 for $\frac{5-\sqrt{2}}{23}+\frac{5+\sqrt{2}}{23}$ allow 3 only for 10/23	5
9	(i) $n = 2m$ $3n^2 + 6n = 12m^2 + 12m$ or $= 12m(m + 1)$ (ii) showing false when n is odd e.g. $3n^2 + 6n$ = odd + even = odd	M1 M2 B2	or any attempt at generalising; M0 for just trying numbers <u>or</u> M1 for $3n^2 + 6n = 3n(n + 2) = 3 \times$ even × even <u>and</u> M1 for explaining that 4 is a factor of even × even <u>or</u> M1 for 12 is a factor of $6n$ when n is even <u>and</u> M1 for 4 is a factor of n^2 so 12 is a factor of $3n^2$ or $3n(n + 2)$ =3 × odd × odd = odd or counterexample showing not always true; M1 for false with partial explanation or incorrect calculation	5

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Section B

10	**i**	correct graph with clear asymptote $x = 2$ (though need not be marked)	G2	G1 for one branch correct; condone (0, − ½) not shown SC1 for both sections of graph shifted two to left		
		(0, − ½) shown	G1	allow seen calculated	3	
	ii	11/5 or 2.2 o.e. isw	2	M1 for correct first step	2	
	iii	$x = \frac{1}{x-2}$	M1	or equivs with *y*s		
		$x(x - 2) = 1$ o.e.	M1			
		$x^2 - 2x - 1$ [= 0]; ft their equiv eqn	M1	or $(x - 1)^2 - 1 = 1$ o.e.		
		attempt at quadratic formula	M1	or $(x - 1) = \pm\sqrt{2}$ (condone one error)		
		$1 \pm\sqrt{2}$ cao	A1			
		position of points shown	B1	on their curve with $y = x$ (line drawn or $y = x$ indicated by both coords); condone intent of diagonal line with gradient approx 1through origin as $y = x$ if unlabelled	6	11
11	**i**	$(x - 2.5)^2$ o.e.	M1			
		$- 2.5^2 + 8$	M1	for clear attempt at -2.5^2		
		$(x - 2.5)^2 + 7/4$ o.e.	A1	allow M2A0 for $(x - 2.5) + 7/4$ o.e. with no $(x - 2.5)^2$ seen		
		min $y = 7/4$ o.e. [so above x axis] or commenting $(x - 2.5)^2 \geq 0$	B1	ft, dep on $(x - a)^2 + b$ with b positive; condone starting again, showing $b^2 - 4ac < 0$ or using calculus	4	
	ii	correct symmetrical quadratic shape	G1			
		8 marked as intercept on y axis	G1	or (0, 8) seen in table		
		tp (5/2, 7/4) o.e. or ft from (i)	G1		3	
	iii	$x^2 - 5x - 6$ seen or used	M1	or $(x - 2.5)^2$ [> or =] 12.25 or ft $14 - b$		
		−1 and 6 obtained	M1	also implies first M1		
		$x < -1$ and $x > 6$ isw or ft their solns	M1	if M0, allow B1 for one of $x < -1$ and $x > 6$	3	
	iv	min = (2.5, − 8.25) or ft from (i)	M1	or M1 for other clear comment re translated 10 down and A1 for referring to min in (i) or graph in (ii); or M1 for correct method for solving $x^2 - 5x - 2 = 0$ or using $b^2 - 4ac$ with this and A1 for showing real solns eg $b^2 - 4ac = 33$; allow M1A0 for valid comment but error in −8.25 ft; allow M1 for showing y can be neg eg (0, −2) found and A1 for correct conclusion		
		so yes, crosses	A1		2	12

12	**i**	$(x-4)^2 - 16 + (y-2)^2 - 4 = 9$ o.e. rad = $\sqrt{29}$	M2 B1	M1 for one completing square or for $(x-4)^2$ or $(y-2)^2$ expanded correctly <u>or</u> starting with $(x-4)^2 + (y-2)^2 = r^2$: M1 for correct expn of at least one bracket and M1 for $9 + 20 = r^2$ o.e. <u>or</u> using $x^2 - 2gx + y^2 - 2fy + c = 0$ M1 for using centre is (g, f) [must be quoted] and M1 for $r^2 = g^2 + f^2 - c$	3	
	ii	$4^2 + 2^2$ o.e = 20 which is less than 29	M1 A1	allow 2 for showing circle crosses x axis at −1 and 9 or equiv for y (or showing one positive; one negative); 0 for graphical solutions (often using A and B from (iii) to draw circle)	2	
	iii	showing midpt of AB = (4, 2) and showing AB = $2\sqrt{29}$ or showing AC or BC = $\sqrt{29}$ or that A or B lie on circle <u>or</u> showing both A and B lie on circle (or AC = BC = $\sqrt{29}$), and showing AB = $2\sqrt{29}$ or that C is midpt of AB or that C is on AB or that gradients of AB and AC are the same or equiv. <u>or</u> showing C is on AB and showing both A and B are on circle or AC = BC = $\sqrt{29}$	2 2 2 2 2 2	in each method, two things need to be established. Allow M1 for the concept of what should be shown and A1 for correct completion with method shown allow M1A0 for AB just shown as $\sqrt{116}$ not $2\sqrt{29}$ allow M1A0 for stating mid point of AB = (4,2) without working/method shown NB showing AB = $2\sqrt{29}$ and C lies on AB is not sufficient – earns 2 marks only if M0, allow SC2 for accurate graph of circle drawn with compasses and AB joined with ruled line through C.	4	
	iv	grad AC or AB or BC = −5/2 o.e. grad tgt = −1/their grad AC tgt is $y - 7$ = their $m\,(x - 2)$ o.e. $y = 2/5\,x + 31/5$ o.e.	M1 M1 M1 A1	may be seen in (iii) but only allow this M1 if they go on to use in this part allow for $m_1m_2 = -1$ used eg y = their $mx + c$ then (2, 7) subst; M0 if grad AC used condone $y = 2/5x + c$ and $c = 31/5$ o.e.	4	13

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General Comments

Many of the candidates were able to make a strong start in the first few questions of this paper, with some of the weaker candidates scoring over half their marks on questions 1 to 5. This resulted in fewer scores under 10 marks on the paper compared with last January. At the other end of the scale, the examiners were pleased to see the strong candidates gaining some very high marks, although a few question parts remained challenging for such candidates, with relatively few candidates realising that two conditions needed to be shown in question 12(iii), for example.

Errors in basic arithmetic were common in some questions, including from the able candidates, such as 3.4 − 5 = 1.6 or −2.4 in question 4, 25 + 8 = 32 in question 11(iv) and errors in the method for adding fractions in question 8. In this non-calculator paper, it is always sad to see marks lost needlessly for low-level work.

The use of graph paper remains a problem for the candidates of some centres who still issue this. Where it is issued, candidates tend to use it, and spend time drawing accurate graphs when a sketch is what is required. One candidate who drew a sketch in the body of the script and gained marks for it commented ‘I have done this again on graph paper in case I need to, but I don’t think I do’, evidence of the confusion caused to candidates by supplying the graph paper. The examiners are grateful to the many centres who do not issue it.

There was some evidence of the paper being a little long, with a few examiners feeling that there were some rushed attempts at question 12. This was sometimes due to long methods being used earlier.

Comments on Individual Questions

Section A

1) Many candidates earned full marks here. A few weaker candidates used addition/subtraction instead of multiplication/division as their first step but then usually earned the last mark for a correct follow through from their v^2.

2) Stronger candidates usually had no problem here and there were many wholly correct answers. Some spoiled their efforts by attempting to simplify further after having achieved the right expression. Factors of $x^2 - 1$ seemed to elude quite a number of candidates, especially the weaker ones, with $x(x - 1)$ being a common error. Many were able to factorise the numerator correctly. Some of the weaker candidates attempted to use the quadratic formula to help them factorise the numerator.

3) Most knew $\left(\frac{1}{4}\right)^0 = 1$, although a few, as expected, thought it was 0. In the second part, there were many correct answers and of those who did not earn all three marks, many were able to deal correctly with at least one aspect of the powers.

4) Many were able to find x correctly but there were errors in finding y, as noted in the general comments. Direct substitution for y was not always the preferred method, many candidates opting to manipulate the equations and go for a method of elimination which, in the main, led to correct answers. In some cases, however, this resulted in marks lost because of sign or arithmetic errors, especially when dealing with double negatives.

5) Better candidates had no problem determining the correct gradient. Quite a few candidates "lost" the negative sign and some weaker candidates merely stated the coefficient of *x* in the given equation as the gradient. In the second part, most successful attempts started from using $y = mx + c$ with the same gradient and stating or finding that $c = 12$, although a few used the condition for perpendicular lines in error. Relatively few candidates attempted the $4x + 5y = K$ argument or a step method using the gradient. Quite a few of those candidates who failed to score, fully or at all, in part (i) were able to benefit from the two method marks in this part by following through with their gradient and using $y = 0$ in their equation. The final mark was again an opportunity for poor arithmetic and algebraic manipulation: many candidates who started correctly from an equation such as $\frac{4}{5}x = 12$ did not proceed easily and correctly to $4x = 60$ or $\frac{x}{3} = 5$ and hence $x = 15$ but rather to statements such as $0.8x = 12$ so $x = 12/0.8$ and then did not know how to proceed or made errors in doing so.

6) Most candidates correctly attempted to use f(2) = 3 and many of these successfully found the correct value of *k*. Those who attempted long division were usually less successful, often only gaining one method mark.

7) The first part was poorly answered. Of those who were familiar with the notation and knew what to do, a disappointing number correctly cancelled to find $\frac{8\times7\times6}{3\times2\times1}$ but then worked it out as $\frac{336}{6}$, with long calculations clearly seen. They often reached the correct answer, but clearly much time had been wasted. Many started again in the second part, even those who had successfully calculated 56 earlier. Full expansions were seen from some. Many reached $56 \times 1^5 \times (-\tfrac{1}{2}x)^3$ but then did not cube ½ or lost the negative sign.

8) There was a varied response to the first part, with strong candidates quickly reaching the correct answer but weaker candidates getting muddled in their attempts to simplify $\sqrt{48}$, as expected. A correct stage of $4\sqrt{3} + \sqrt{3}$ was sometimes followed by $4 + 2\sqrt{3}$. In the second part, most had some idea of what to do, but weaker candidates often made errors in their method for adding fractions, such as adding numerators and denominators, perhaps having first found a common denominator, so that answers such as $\frac{1}{23}$, $\frac{10}{46}$ and $\frac{1}{10}$ were seen, as well as the usual arithmetic errors in working out $5^2 - \left(\sqrt{2}\right)^2$. Some rationalised the denominator of just one fraction, or of each fraction separately before adding them. Stronger candidates usually had no problem.

9) In the first part, those who used $n = 2k$ were able elegantly to gain their three marks, but few candidates used this approach. Some worked with $3n(n + 2)$ or the given expression and used odd and even number arguments whilst others divided by 12 and showed that $\frac{n^2}{4} + \frac{n}{2}$ was an integer when *n* was even. However, many simply substituted some even numbers and jumped to the conclusion, gaining no marks, with some claiming proof by exhaustion! The marks in the second part were more easily gained, and weaker candidates were often able to use a counterexample correctly here. There was considerable confusion of factors and multiples in explanations throughout the question.

Section B

10) (i) This part was done poorly - only a minority of candidates appreciated that a translation 2 to the right was what was required, although some successfully found $(0, -0.5)$ as the y-intercept independently.

(ii) Many candidates solved this equation successfully, although some made algebraic or arithmetic errors.

(iii) Most candidates equated the two expressions for y and made some progress towards solving the resulting equation. Knowledge of the quadratic formula was better than last time, with fewer candidates making errors in remembering it. As expected, many did not simplify the final answer correctly, but the correct answer was seen encouragingly often. Candidates who sensibly drew the line $y = x$ on their part (i) sketch and identified the intersections as the points required obtained the final mark in this section for doing so; those who omitted the $y = x$ line needed to show that the x and y coordinates were the same and often did not.

11) This question was accessible to most candidates, even if they were not able to do the first part. There were some centres who were clearly well versed in completing the square – others less so but they were still able to proceed with parts (ii) to (iv).

(i) Most candidates were able to earn at least the first method mark here. Further progress was limited for many candidates by their inability to find 2.5^2. For those who at least showed that they were trying to do this calculation, there was the possibility of the next method mark; but many showed no working, so were not able to gain credit. The most common error was to believe 2.5^2 to be 4.25 and so a very common wrong answer was $(x – 2.5)^2 + 3.75$. It was surprising that so many candidates did not use their result to explain why the graph was above the axis, or attempted it and failed to explain clearly. Many resorted to the algebraic technique of considering the discriminant of the equation where $y = 0$.

(ii) This part was attempted quite well, but a large number of candidates failed to make the intercept on the y-axis and the position of the turning point clear. A few candidates answered the final stage of part (i) here, without realising it. Some did not realise the relevance of the work they had done in part (i) and started again, using symmetry or calculus to find the coordinates of the turning point.

(iii) Most candidates successfully rearranged the inequality to produce $x^2 - 5x - 6 > 0$.The majority of these managed to arrive at the values of 6 and -1. However it was all too common to see final answers of $-1 < x < 6$ or $-1 > x > 6$. A very small number of candidates produced factors involving 3 and 2 and some were able to pick up the final mark on a follow through basis. A few candidates tried a numerical approach which rarely reached a conclusion, though one or two candidates did conclude that $x > 6$.

(iv) A variety of methods were applied here with varying degrees of success. Having written down an expression for $f(x) -10$, a number of candidates spotted that the graph would now cut the y-axis at (0 , 2) - a method requiring little working. Others tried to show that the minimum of the new graph had a negative y coordinate, too often hampered by their inability to subtract 10 from a number involving fractions! Once again many candidates used the discriminant method, and in this part seemed more secure in their conclusion. A few candidates used sketch graph, but not all of them quantified a strategic point. One or two candidates used the straightforward approach of showing that there were some points on the curve above the axis, and some below.

12) (i) Many candidates failed to have a systematic approach to changing the form of the equation to show that it was a circle. Those completing the square, did not often realise where the 9 fitted in. Candidates who started from the given equation and completed the square were mostly successful. Those who started from $(x - 4)^2 + (y - 2)^2$ often got in a muddle with the signs of the constant terms and many only obtained one mark. Many of the weaker candidates thought that the radius was 9 or 3. A few candidates used the *g f* version of the general equation – with mixed results, depending on how much information was quoted.

(ii) This was the least well done part of the question; some candidates made no attempt at it. Others gained a method mark for working out $4^2 + 2^2$, although some were unable to complete the argument from this, with some not knowing what to do and others having used a wrong radius such as 3. Those who put (0, 0) into the original given equation often did not appreciate what they were doing, ended up with 0 = 9 and did nothing further, gaining no credit. A small number of candidates successfully showed that the origin was on a chord within the circle (usually by finding $x = -1$ or 9 when $y = 0$).

(iii) The modal score was 2 out of the 4 marks, with most candidates showing one of the two criteria necessary for AB to be a diameter, and few realising that two were required. The most popular method was to try to show that the length of AB was double the radius, which caused problems if they had the radius wrong, although many got as far as the length being $\sqrt{116}$. A large number of candidates showed that the midpoint of AB was C, with a few using vector methods to do this. Those who started by showing that both A and B were on the circle generally realised that something else was needed. Some candidates with the wrong radius in part (i), having found answers in (iii) such as AB = $\sqrt{116}$, or AC = $\sqrt{29}$, were able to go back to part (i) and find their errors there.

(iv) Many candidates were able to handle the coordinate geometry of straight lines, including a good proportion of those who had proved weak elsewhere. Sign errors were common in finding the gradient of AB, but most knew the rule for perpendicular gradients and used it well. A few candidates tried to differentiate the circle equation, but were rarely successful. There were many arithmetic errors in simplifying the equation of the line and some did not give an answer in the requested $y = mx + c$ form, so that loss of the final accuracy mark was common.